The Murder Mystery Method to Solve Exact Differential Equations

http://homepage.cem.itesm.mx/lgomez/

- If you are using explorer for Windows, then you can see these slides in full-screen format
- It is IMPORTANT to see these slides in full-screen format, otherwise you might not be able to read the mathematical formulas
- When you are inside the full-screen format, click with the mouse's left button on the slide in order to go to next slide
- In order to start, click the full-screen icon (it is usually located in the lower right corner of this window).

Reference

- This document is based on the work by Tevian Dray and Corinne A Manogue:

- Tevian Dray and Corinne A Manogue, **The murder mystery method for determining whether a vector field is conservative**; *The College Mathematics Journal*; May 2003; **34**, 3; Academic Research Library; pages 228-231

Creating a Differential Equation from a Function of Two Variables (1)

Consider the function

$$F(x,y) = 3xy^2 + x^4 + y^2$$

Creating a Differential Equation from a Function of Two Variables (2)

Consider the function

$$F(x,y) = 3xy^2 + x^4 + y^2$$

The differential change of this function is given by

$$dF = \frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial y}dy$$

$$dF = \left(3y^2 + 4x^3\right)dx + \left(6xy + 2y\right)dy$$

Creating a Differential Equation from a Function of Two Variables (3)

Assume we move in a trajectory where the function has a constant value

$$F(x,y) = 3xy^2 + x^4 + y^2 = C$$

Creating a Differential Equation from a Function of Two Variables (4)

Assume we move in a trajectory where the function has a constant value

$$F(x,y) = 3xy^2 + x^4 + y^2 = C$$

Then the differential change of the function must be zero:

$$\left(3y^2 + 4x^3\right)dx + \left(6xy + 2y\right)dy = 0$$

Creating a Differential Equation from a Function of Two Variables (5)

So we start with a relationship between *x* and *y*

$$F(x,y) = 3xy^2 + x^4 + y^2 = C$$

And we obtain a differential equation

$$\left(3y^2 + 4x^3\right)dx + \left(6xy + 2y\right)dy = 0$$

We want to do the inverse procedure

We want to be able to do the inverse procedure, start with the differential equation

$$\left(3y^2+4x^3\right)dx+\left(6xy+2y\right)dy=0$$

And obtain as an answer the relationship between *x* and *y*

The mystery we have to solve

Therefore this is the mystery we have to solve:

If we have an equation in the form $M\mathrm{d}x+N\mathrm{d}y=0$; is there a function $F(x,y)$ such that its differential change is $M\mathrm{d}x+N\mathrm{d}y$?

- If the answer is YES, then the solution of the differential equation is the relationship $F(x,y)=C$ (the equation is called "exact")
- If the answer is NO, then we have to use another method to solve the differential equation (the equation is called "nonexact")

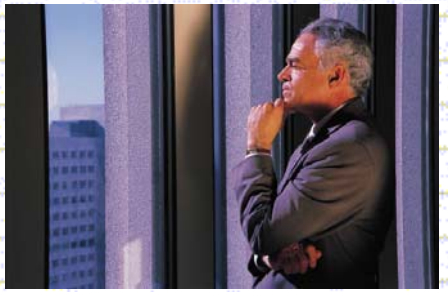
You are the Detective!

- A crime has been committed by the unknown murderer *F*; you are the detective, and your job is to find the identity of *F* by interrogating the witnesses

The murderer *F* and the witnesses *M* and *N*

The murderer *F*

Witness *M*



Witness *N*

Interrogate witnesses

Witness *M*:
"He was wearing green jacket and red pants"

Witness *N*:
"He was wearing red pants and brown shoes"

If clues are consistent, then you know what the murderer *F* was wearing

Witness *M*:
"He was wearing green jacket and red pants"

Witness *N*:
"He was wearing red pants and brown shoes"

Clues are consistent, you know what the murderer *F* was wearing

If clues are not consistent, then you do not know what the murderer *F* was wearing

Witness *M*:
"He was wearing green jacket and yellow pants"

Witness *N*:
"He was wearing violet pants and brown shoes"

Clues are NOT consistent: In this case you do **not** trust the witnesses and you do not have your murderer

Murder Mystery Method Applied to Identify and Solve Exact Differential Equations

- Witnesses M and N are in the differential equation: $M\mathrm{d}x+N\mathrm{d}y=0$
- To interrogate them is to integrate them
- Clues that have both x and y must have been seen by the two witnesses, otherwise, they are not consistent and there is no murderer F (maybe it was a suicide?).

Example 1

- Solve: $(3y^2+4x^3\ )dx+(6xy+2y)dy=0$

Example 1: Interrogation (Integration)

- Solve: $(3y^2+4x^3)dx+(6xy+2y)dy=0$

Witness *M*:

$$\int \left(3y^2+4x^3\right)dx = 3xy^2+x^4$$

Witness *N*:

$$\int \left(6xy+2y\right)dy = 3xy^2+y^2$$

Example 1: Check consistency

- Solve: $(3y^2+4x^3)dx+(6xy+2y)dy=0$

Witness *M*:

$$\int \left(3y^2+4x^3\right)dx = 3xy^2+x^4$$

Witness *N*:

$$\int \left(6xy+2y\right)dy = 3xy^2+y^2$$

The clue that contains the two variables ($3xy^2$) was seen by the two witnesses: Therefore we **do** trust them and we have our **murderer** (the function whose differential is the left side of the equation):

$$F(x,y) = 3xy^2+x^4+y^2$$

Example 1: Final Solution

- Solve: $(3y^2+4x^3)dx+(6xy+2y)dy=0$

Witness *M*:

$$\int (3y^2+4x^3)dx = 3xy^2+x^4$$

Witness *N*:

$$\int (6xy+2y)dy = 3xy^2+y^2$$

The clue that contains the two variables ($3xy^2$) was seen by the two witnesses: Therefore we **do** trust them and we have our murderer (the function whose differential is the left side of the equation):

$$F(x,y) = 3xy^2+x^4+y^2$$

The implicit solution of the differential equation is

$$\boxed{3xy^2+x^4+y^2=C}$$

Example 2

- Solve: $(y+x)\mathrm{d}x+(y)\mathrm{d}y=0$

Example 2: Interrogate (Integrate)

- Solve: $(y+x)dx+(y)dy=0$

Witness *M*:

$$\int (y+x)dx = xy + \frac{x^2}{2}$$

Witness *N*:

$$\int (y)dy = \frac{y^2}{2}$$

Example 2: Check consistency

- Solve: $(y+x)dx+(y)dy=0$

Witness *M*:

$$\int (y+x)dx = xy + \frac{x^2}{2}$$

Witness *N*:

$$\int (y)dy = \frac{y^2}{2}$$

The clue that contains the two variables (xy) was seen only by one of the two witnesses: Therefore we do **not** trust them, and we do **not** have our murderer

Example 2: Final Solution

- Solve: $(y+x)dx+(y)dy=0$

Witness *M*:

$$\int (y+x)dx = xy + \frac{x^2}{2}$$

Witness *N*:

$$\int (y)dy = \frac{y^2}{2}$$

The clue that contains the two variables (*xy*) was seen only by one of the two witnesses: Therefore we do **not** trust them, and we do **not** have our murderer

This equation is Nonexact, it has to be solved by another method

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