

The Murder Mystery Method to Solve Exact Differential Equations

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- If you are using explorer for Windows, then you can see these slides in full-screen format
- It is IMPORTANT to see these slides in full-screen format, otherwise you might not be able to read the mathematical formulas
- When you are inside the full-screen format, click with the mouse's left button on the slide in order to go to next slide
- In order to start, click the full-screen icon (it is usually located in the lower right corner of this window).

Reference

- This document is based on the work by Tevian Dray and Corinne A Manogue:
- Tevian Dray and Corinne A Manogue, **The murder mystery method for determining whether a vector field is conservative**; *The College Mathematics Journal*; May 2003; **34**, 3; Academic Research Library; pages 228-231

Creating a Differential Equation from a Function of Two Variables (1)

Consider the function

$$F(x, y) = 3xy^2 + x^4 + y^2$$

Creating a Differential Equation from a Function of Two Variables (2)

Consider the function

$$F(x, y) = 3xy^2 + x^4 + y^2$$

The differential change of this function is given by

$$dF = \frac{\partial F}{\partial x} dx + \frac{\partial F}{\partial y} dy$$

$$dF = (3y^2 + 4x^3)dx + (6xy + 2y)dy$$

Creating a Differential Equation from a Function of Two Variables (3)

Assume we move in a trajectory where the function has a constant value

$$F(x, y) = 3xy^2 + x^4 + y^2 = C$$

Creating a Differential Equation from a Function of Two Variables (4)

Assume we move in a trajectory where the function has a constant value

$$F(x, y) = 3xy^2 + x^4 + y^2 = C$$

Then the differential change of the function must be zero:

$$(3y^2 + 4x^3)dx + (6xy + 2y)dy = 0$$

Creating a Differential Equation from a Function of Two Variables (5)

So we start with a relationship between x and y

$$F(x, y) = 3xy^2 + x^4 + y^2 = C$$

And we obtain a differential equation

$$(3y^2 + 4x^3)dx + (6xy + 2y)dy = 0$$

We want to do the inverse procedure

We want to be able to do the inverse procedure, start with the differential equation

$$(3y^2 + 4x^3)dx + (6xy + 2y)dy = 0$$

And obtain as an answer the relationship between x and y

The mystery we have to solve

Therefore this is the mystery we have to solve:

If we have an equation in the form $Mdx+Ndy=0$; is there a function $F(x,y)$ such that its differential change is $Mdx+Ndy$?

- If the answer is YES, then the solution of the differential equation is the relationship $F(x,y)=C$ (the equation is called "exact")
- If the answer is NO, then we have to use another method to solve the differential equation (the equation is called "nonexact")

You are the Detective!

- A crime has been committed by the unknown murderer F ; you are the detective, and your job is to find the identity of F by interrogating the witnesses

The murderer F and the witnesses M and N

The murderer F



Witness M



Witness N



Interrogate witnesses

Witness *M*:

"He was wearing
green jacket and red
pants"

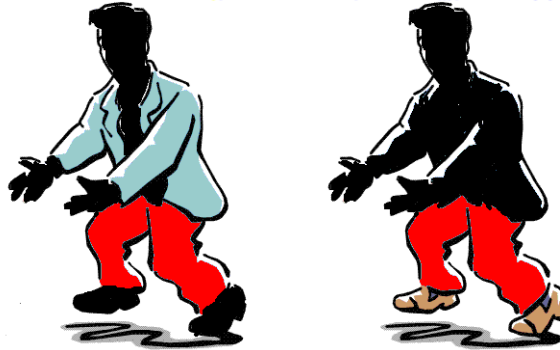


Witness *N*:

"He was wearing red
pants and brown
shoes"

If clues are consistent, then you know what the murderer F was wearing

Witness M :
"He was wearing
green jacket and red
pants"



Witness N :
"He was wearing red
pants and brown
shoes"

Clues are consistent,
you know what the
murderer F was
wearing



If clues are not consistent, then you do not know what the murderer *F* was wearing

Witness *M*:
"He was wearing green jacket and yellow pants"



Witness *N*:
"He was wearing violet pants and brown shoes"

Clues are NOT consistent: In this case you do **not** trust the witnesses and you do not have your murderer

Murder Mystery Method Applied to Identify and Solve Exact Differential Equations

- Witnesses M and N are in the differential equation: $Mdx + Ndy = 0$
- To interrogate them is to integrate them
- Clues that have both x and y must have been seen by the two witnesses, otherwise, they are not consistent and there is no murderer F (maybe it was a suicide?).

Example 1

□ Solve: $(3y^2 + 4x^3)dx + (6xy + 2y)dy = 0$

Example 1: Interrogation (Integration)

□ Solve: $(3y^2 + 4x^3)dx + (6xy + 2y)dy = 0$

Witness *M*:

$$\int (3y^2 + 4x^3)dx = 3xy^2 + x^4$$



Witness *N*:

$$\int (6xy + 2y)dy = 3xy^2 + y^2$$

Example 1: Check consistency

□ Solve: $(3y^2 + 4x^3)dx + (6xy + 2y)dy = 0$

Witness *M*:



Witness *N*:

$$\int (3y^2 + 4x^3)dx = 3xy^2 + x^4$$

$$\int (6xy + 2y)dy = 3xy^2 + y^2$$

The clue that contains the two variables ($3xy^2$) was seen by the two witnesses: Therefore we **do** trust them and we have our **murderer** (the function whose differential is the left side of the equation):

$$F(x, y) = 3xy^2 + x^4 + y^2$$

Example 1: Final Solution

□ Solve: $(3y^2 + 4x^3)dx + (6xy + 2y)dy = 0$

Witness *M*:



Witness *N*:

$$\int (3y^2 + 4x^3)dx = 3xy^2 + x^4$$

$$\int (6xy + 2y)dy = 3xy^2 + y^2$$

The clue that contains the two variables ($3xy^2$) was seen by the two witnesses: Therefore we **do** trust them and we have our murderer (the function whose differential is the left side of the equation):

$$F(x, y) = 3xy^2 + x^4 + y^2$$

The implicit solution of the differential equation is

$$3xy^2 + x^4 + y^2 = C$$

Example 2

□ Solve: $(y+x)dx+(y)dy=0$

Example 2: Interrogate (Integrate)

□ Solve: $(y+x)dx+(y)dy=0$

Witness M :

$$\int (y+x)dx = xy + \frac{x^2}{2}$$



Witness N :

$$\int (y)dy = \frac{y^2}{2}$$

Example 2: Check consistency

□ Solve: $(y+x)dx+(y)dy=0$

Witness M :

$$\int (y+x)dx = xy + \frac{x^2}{2}$$



Witness N :

$$\int (y)dy = \frac{y^2}{2}$$

The clue that contains the two variables (xy) was seen only by one of the two witnesses: Therefore we do **not** trust them, and we do **not** have our murderer

Example 2: Final Solution

□ Solve: $(y+x)dx+(y)dy=0$

Witness M :

$$\int (y+x)dx = xy + \frac{x^2}{2}$$



Witness N :

$$\int (y)dy = \frac{y^2}{2}$$

The clue that contains the two variables (xy) was seen only by one of the two witnesses: Therefore we do **not** trust them, and we do **not** have our murderer

This equation is Nonexact, it has to be solved by another method

Reference

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